Summer Research Experience Undergraduate Program in Mathematics at Tuskegee University (NREUP), Supported by the National Science Foundation and the National Security Agency. June 8, 2009 through July 31, 2009.

A Summer Research Project on Applying Linear Algebra in Different Areas of Differential, Modern Geometry and Knot Theory
Supported by National Research for Undergraduate Program (NREUP)

Research Faculty

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Project summary:

Again the faculty at Tuskegee University seeks to encourage under-represented minority students to plan, prepare and pursue the graduate degrees in mathematics and/or related fields through research and scholarly achievement. The undergraduate students chosen for the 2009 Summer NREUP will concentrate on problems embedded in the areas of Linear Algebra, Differential Geometry, and Modern Geometry.

Project 1: **Aspects of Linear Algebra in Differential Geometry**

Objective:

This project is designed to introduce the undergraduate students to the use of various aspects of linear algebra in solving problems in differential geometry. The students will first learn about the relationships between the n-spheres, and the Euclidean space. For instance, they will show that the \((n + 1)\) Euclidean space minus the origin is exactly the product of the n-sphere and the real line. They will learn about tangent lines, and planes to the n-sphere, as well as normal lines, and the concept of tangent and normal bundles. They will proceed to learn about some special subgroups of the \(n \times n\) matrices. In particular they will learn about \(O(n)\), the group of orthogonal matrices, and \(SO(n)\), the subgroup of \(O(n)\), consisting of those matrices with determinant 1, and their relationship to the n-sphere. They will prove that the n-sphere minus a point is exactly the n-dimensional Euclidean space. They will then learn about manifolds, and proceed to work on several interesting problems. They will show that \(SO(n)\) is diffeomorphic to the product of the \((2n - 1)\)-sphere and \(SO(2n-1)\) only when \(n = 1, 2, 4\). They will be given several other problems relating linear algebra, geometry and differential equations to explore. More precisely, they can explore flows induced by the maps from the n-spheres to \(SO(2n)\), for \(n = 1, 2, 4\).

Outcome:

At the end of the project the students should be familiar with the basic tools of linear algebra that are used in differential geometry. They will be able to solve some challenging problems using simple methods. In particular they will see how different branches of Mathematics are very well connected.
Project 2: Expressing a Linear Transformation as a Product of Rotations and a Simple Affine Map

Objective:

The aim of this project is to show that a linear transformation can be represented as a product of rotations and a simple affine map represented in matrix form by $A = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$. The student will have an opportunity to show that any nonsingular matrix (or transformation) may be decomposed, and that the given transformation completely determines the parameters “a” and “b” of matrix A. This decomposition must be of the form $AB = PDP^{-1}$ where $\det P = 1$. It is expected that the student will discover that any linear transformation $g$ may be represented as the product $g = h_\alpha Ah_\beta$ where $A$ is the transformation $x' = ax$, $y' = by$ and $h_\alpha$ represents a rotation through an angle $\alpha$. This project will require a background in linear algebra.

Outcome:

At the end of the project the student will have some general knowledge of the relationships between geometric transformations in geometry and certain aspects of linear algebra. It is expected that the student will gain knowledge about the applicants of mathematics studied in an elementary linear algebra course. Also, it is expected that the student will become interested in exploring other relationships in transformational geometry.
Project 3: Derivation of Heron-like Formula for Finding the Area of a Triangle Expressed in Terms of Its Medians

Objective:

This is a continuation of a similar project done by a student who participated in the Summer 2008 Program at Tuskegee. However, the Summer 2008 project involved deriving a Heron-like Formula for the area of a triangle expressed in terms of its altitudes. The proposed project for Summer 2009 will involve doing the problem using medians. The in depth ideas to be explored in this project are not normally covered in a first year Modern Geometry course due to a time constraint in trying to cover a variety of topics. The student will first begin by studying the four or more different ways one can prove Heron’s Formula for finding the area of a triangle. Also, the student will explore in depth Stewart’s Formula for finding the lengths of the three medians of a triangle in terms of the sides. Also, a review of various proofs of Ceva’s Theorem for proving concurrency of lines drawn from the median to the opposite side of a triangle will be done. Armed with this background information, the student will proceed to prove (find) a formula for the sides of a triangle in terms of its medians. The researcher will determine whether these lengths can be given arbitrarily to determine a triangle or whether the triangle is uniquely determined. Next, the researcher will establish a Heron-like formula for the area of a triangle in terms of its three altitudes. Additional relations concerning with area of a triangle in terms of medians will be derived.

Outcome:

At the conclusion of the project, the student will be familiar with some selected properties of a triangle in terms of finding its area using uncommon techniques. The student will learn how to do some of these derivations synthetically as well as analytically. It is expected that the student’s interest in geometry will be enhanced by studying and deriving some of the beautiful properties for finding the area of a triangle using medians. Also, it is expected that the student will become interested in explore other historical relationships in geometry.

Student Researchers:
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3. Deirdre D. Bellamy
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